Techniques and results concerning relationships between modal ∃-logics

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Validity of a modal formula on a Kripke frame is defined as the global truth under any valuation of propositional variables. Similarly we can consider existential validity: a formula A is existentially valid on a frame $\mathcal{F}$ if it is satisfiable under any valuation. That is, for any valuation, there is a world satisfying A. Collection of formulas existentially valid on a class of frames is called the ∃-logic of the respective class. The ∃-logic corresponding to the class of all frames was determined by Humberstone [3], and Zolin [7] posed the question of axiomatizability of ∃-logics corresponding to other classes of frames.

It is known (a result of Tiomkin and Kaminski [6]) that ∃-logics of $S4$-frames and $S5$-frames coincide. The technique used to prove this result can also be used to prove many other similar results. This includes, as we will show, everything that holds between the ∃-logics of the classes of frames corresponding to the fifteen traditional logics [2] (K extended with a subset of $\{B, D, T, 4,5\}$). The essential requirement for applicability of the method is for a logic to have the so-called global finite model property [4], [5], [1].

References

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